Strategic Asset Allocation with Portfolios of Hedge Funds[1]

By Drago Indjic, Fauchier Partners

Summary

What is the best way to construct a portfolio of hedge funds to supplement a portfolio with or without other alternative investments? Is it possible to optimise strategy mix to improve risk profile?

The Wall Street Journal of December 2000 reported that several investment banks advised clients to allocate up to 10% of their strategic portfolio to alternative investments: Schroeder and Julius Baer recommended an equal split between private equity and hedge funds while UBS Warburg advocated 35% allocation with 2/3 held in hedge funds. The Morgan Stanley Dean Witter (MSDW) hedge fund study by Peskin et al (2000) advised to make hedge fund allocations of up to 20% on the basis of liability benchmark criteria and agreed with the UBS Warburg study by Ineichen (2000) that the supply of market inefficiencies causing performance persistence across various hedge fund strategies would continue into the future.

With the benefit of hindsight, it is clear that in 2001 hedge funds significantly outperformed both the public and private equity markets.

Traditionally strategic asset allocation studies have concentrated on problems involving a simple mix of traditional (fundamental) assets and usually only one alternative asset class, for example in Schneeweis et al (2000), Siddiqui (2000) and in most conference presentations.

Today the benefits of adding a fund of hedge funds to traditional strategic portfolio are "yesterday's news" and the problems facing more sophisticated clients are different.

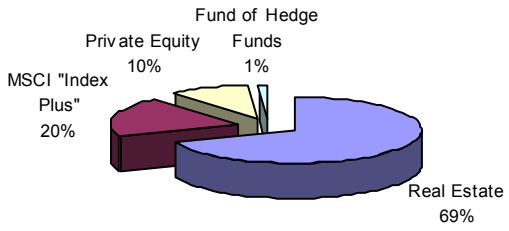
What is the fund of hedge funds benefit in the case of a portfolio that already overweights certain alternative assets including private equity and real estate? Or, what benefit are they to a high-net worth client's strategic portfolio with significant weighting of a single stock?

Our research indicates that the portfolio of hedge funds with a customised mix of hedge fund strategies can offer a superior risk profile than an "off the shelf" fund of hedge funds. Following Schneeweis et al (2000), two extreme cases of customisation benefits are demonstrated in a "**return enhancer**" portfolio that provides pure "alpha" and a "**risk diversifier**" portfolio that minimises the total risk of a strategic "core" portfolio or of a selected basket of asset classes. Several examples of handling various data issues in practice are presented.

Alternative Asset Data in Practice

Consider the strategic portfolio on Fig. 1 that significantly overweighs all alternative asset classes:

Fig. 1 An 100% Alternative Strategic Portfolio



[1]This article was originally submitted to AIMA in December 2001 and as such the data included herein dates back to 2000/2001.

where an investor plans to significantly increase allocation to funds of hedge funds. How should this increase be allocated between different funds of hedge funds? In other words, what is the mix of hedge fund strategies for an "optimal" additional fund of hedge funds?

Depending on the exact nature of the strategic asset allocation objective, several methods of constructing optimal allocation for flexible "satellite" portfolio of hedge funds are feasible but great care should be exercised in data pre-processing. Only a few features of alternative asset indices will be examined here.

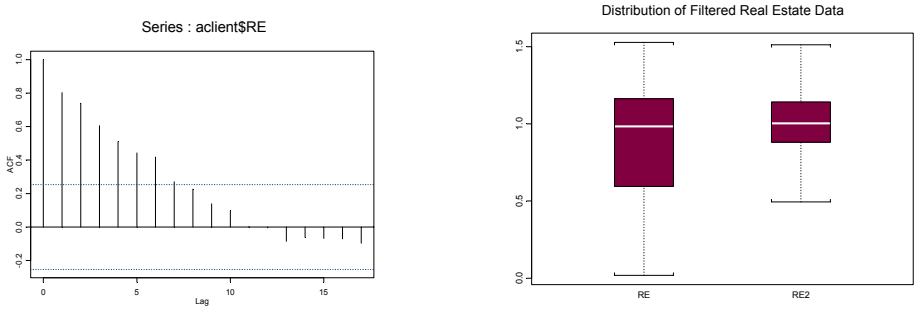
Table 1 lists data, which are used as proxies for strategic asset classes.

Table 1: Strategic asset classes proxies

Code	Asset Class	Index
RE	UK Commercial property	IPD Total Return (All property and Office)
MSCI	"Index + Alpha" Global growth	MSCI World (Gross)
PE	Private Equity	Cambridge Associates US Private Equity
AVAM	"Off the shelf" fund of hedge funds	Fauchier Partners gathered data

The proxies can be further refined to account for sector biases and "alpha" in actual investment vehicles. The main problem in analysis of private equity and real estate indices is their high level of autocorrelation (Fig. 2) and suitable filtering procedure, for example similar to Brooks and Kat (2001) should be performed.

Fig. 2 Real estate index autocorrelation (left) and return distribution before and after filtering (right)



Note that private equity indices (and funds of private equity funds) are even more variable and less transparent than hedge fund indices (see for example de las Heras (2000) comparing WM, EVCA and Venture Economics (but not Wiltshire etc) indices). In addition, private equity indices are quarterly and should be statistically interpolated to provide monthly data values.

The "off the shelf" fund of hedge funds returns are proxied by our own data. However, in order to avoid any portfolio "cherry-picking" biases in analysis we often use the composite which is the average of all portfolios with the same mandate.

After pre-processing of selected datasets, historical mean, variance (Table 2) and correlation matrices (Table 3) are estimated using standard and robust resampling procedures that are less sensitive to outliers and small sample bias.

Table 2: Monthly Return [%] Data Descriptive Statistics (April 1995 – October 2000)

Standard	RE	MSCI	PE	AVAM
Min	0.02	-13.32	-6.29	-6.05
Median	0.98	1.79	2.27	1.20
Mean	0.89	1.56	2.06	1.16
Max	1.53	9.06	5.62	6.10
St.dev	0.39	3.85	1.78	1.99
Robust				
Mean	0.89	1.7	2.1	1.2
St.dev	0.2	3.2	1.4	1.6

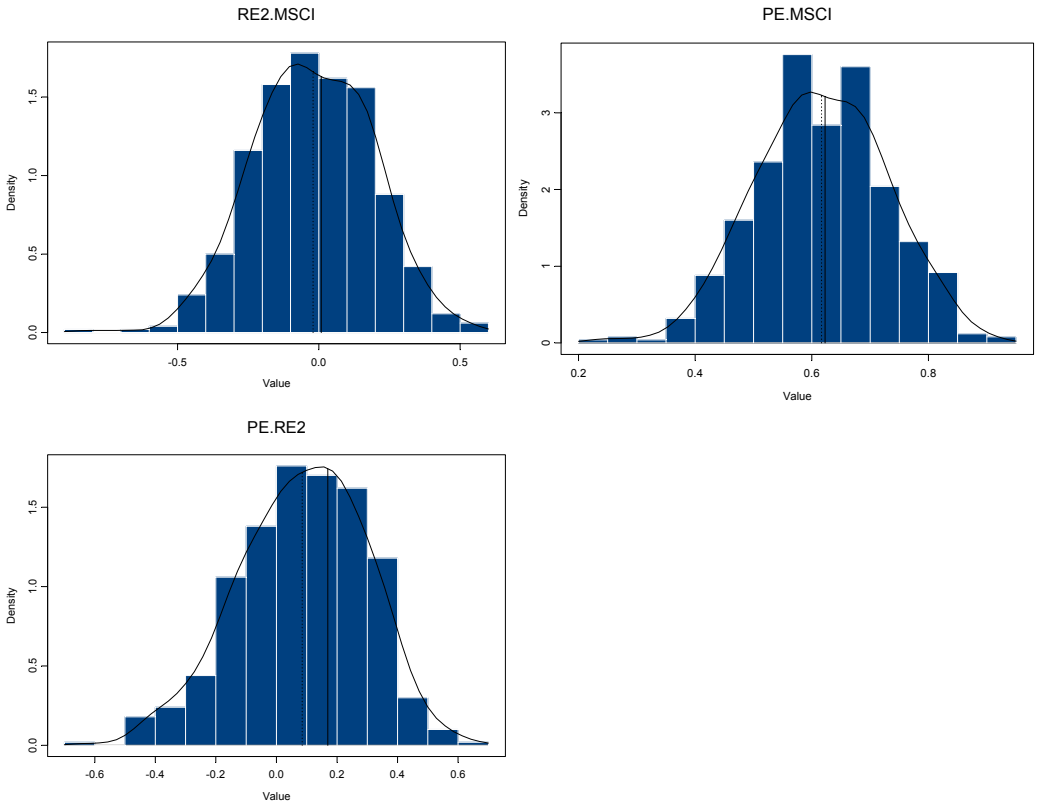
Table 3: Strategic portfolio correlation matrix

	MSCI	AVAM	RE	PE
MSCI	1	0.59	-0.07	0.56
AVAM		1	0.14	0.40
RE			1	0.24
PE				1

Furthermore, Monte Carlo simulation of robust covariance estimation provided insight into correlations between alternative asset classes. The distribution of correlation and estimates of correlation standard errors plotted on Fig. 3 and Fig. 4 were obtained using 500 data replicates.

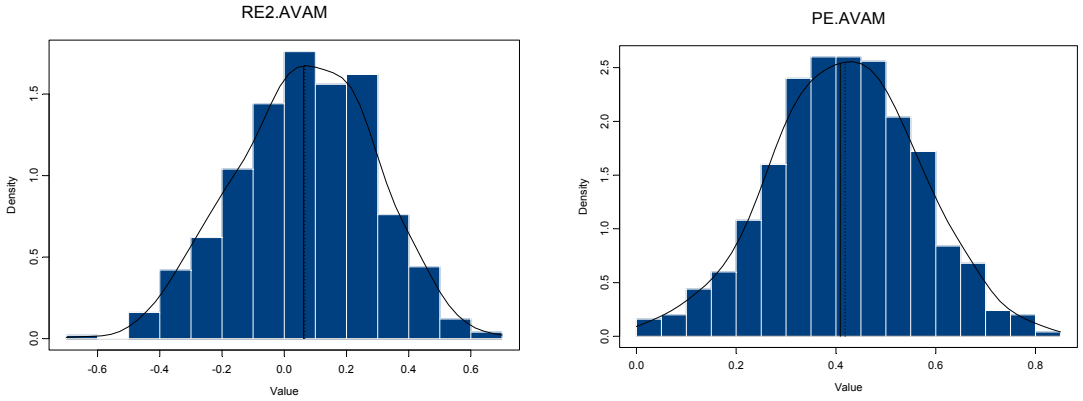
The most important are correlations between alternative assets in strategic portfolio: strongest between private equity and MSCI (~0.6) and small negative between real estate and MSCI (~0.1) and small positive between private equity and real estate (~0.2):

Fig. 3 Alternative assets correlation pairs



Standard funds of hedge funds exhibit positive correlations (~0.4) with equity markets indicating that more flexibility in fund of hedge funds construction is needed to minimise correlations of private equity (~0.4) and real estate (~0.2) with portfolio of hedge funds.

Fig. 4 Funds of hedge funds correlations with alternative assets



Standard optimisation may be performed after carrying out successful data pre-processing.

An alternative to often presented mean-variance optimisation using a general hedge fund index would be to minimise the total (joint) risk of the current portfolio by suitable selection of hedge fund strategies: in other words, to **hedge** principal component(s) of strategic portfolio risk as shown in Example B below. A flexible portfolio of hedge funds can be even customised to be minimally correlated not only to the core portfolio but also to any of its components.

Toward "Gain/Loss" Strategic Efficient Frontier

In investment practice, the strategic benchmarking consultancy is offered by investment consultants including Watson Wyatt, Bacon & Woodrow, Towers Perrin, Aon and others. Regardless of sophisticated dynamical programming widgets that are sometimes used it can be argued that it is more of an art than science.

Starting from the current portfolio allocations, the standard historical mean-variance optimisation (unconstrained, absolute risk/return probability space) produced Table 4 listing three optimal portfolios along the efficient frontier:

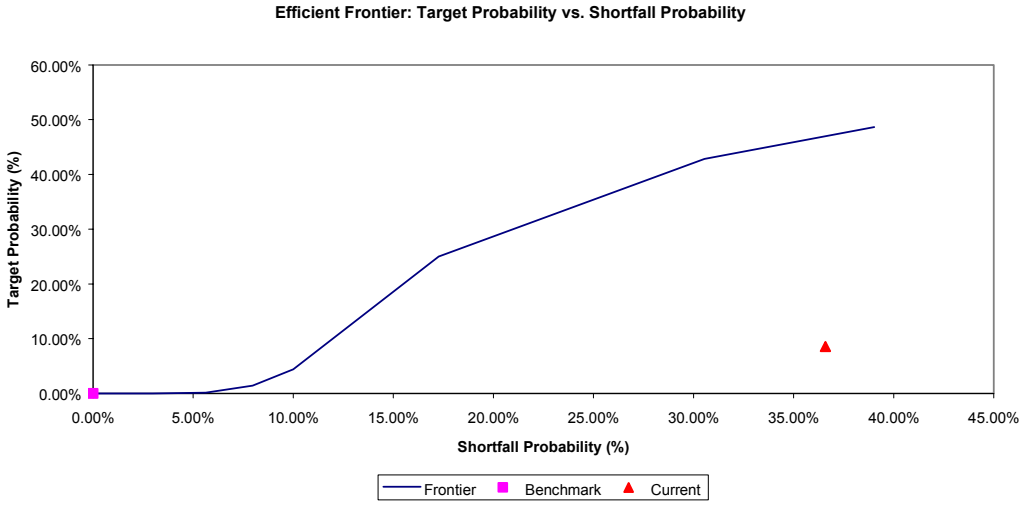
Table 4: Strategic efficient frontiers

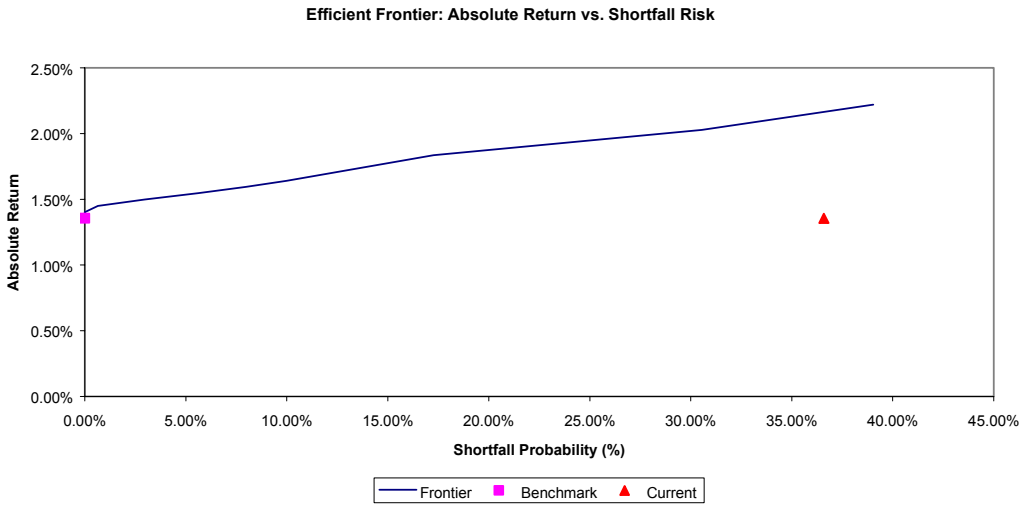
	Traditional	Current	Efficient #1	**Efficient #2**	Efficient #3
Relative Return	0.00%	0.00%	0.12%	**0.25%**	0.37%
Tracking Error	0.00%	0.00%	0.18%	**0.36%**	0.59%
Absolute Return	1.35%	1.356%	1.48%	**1.60%**	1.73%
Absolute Risk	3.16%	2.65%	2.72%	**2.80%**	2.94%
Marginal Sharpe Ratio			0.69	**0.69**	0.47
Conf. of beating target	0.00%	0.00%	0.00%	**1.81%**	14.18%
Risk of Shortfall	0.00%	0.00%	1.88%	**8.33%**	14.53%
Rel. return at conf.level	0.00%	0.00%	-0.11%	**-0.21%**	-0.38%
Single stock	36.00%	36.00%	35.62%	**35.24%**	33.12%
SP500	15.00%	8.00%	13.55%	**19.10%**	24.54%
MSCIXUS	24.00%	10.00%	5.15%	**0.31%**	0.00%
R2000	10.00%	4.00%	4.11%	**4.21%**	7.14%
Merrill	15.00%	15.00%	7.75%	**0.49%**	0.00%
PE	0.00%	7.00%	10.03%	**13.05%**	19.63%
AVAM	0.00%	20.00%	23.79%	**27.59%**	15.56%

An equivalent "traditional" 85%/15% global balanced portfolio with no alternative assets is shown for comparison. Even a non-optimised "off the shelf" portfolio of hedge funds offers **16% reduction in absolute risk** without corresponding reduction in absolute return. The current portfolio historical annualised return is 17.60% and risk 9.18% and the portfolio with annualised return 18.96% (1.45% monthly) and risk 11.19% (3.23% monthly) will be dominated by portfolio #2 and #3. Portfolio #2 would offer a **11% reduction in absolute risk with 18.5% increase in absolute return** – an opportunity that Siddiqui (2000) described as "insurance without premium", but many users of guaranteed (structured) alternative investment products may strongly disagree.

However, traditional benchmark-relative mean-variance optimisation is not in the spirit of alternative investments targeting absolute uncorrelated returns. As a first step toward real absolute returns analysis framework the standard efficient frontier plots are substituted by Fig. 5 plots showing **confidence of beating return target** and **shortfall risk.**

Fig. 5 Alternative representations of efficient frontiers





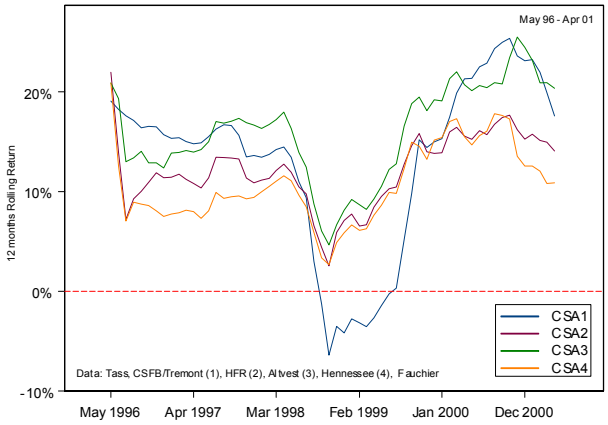
It can be expected that much better framework for analysing absolute return assets would be the gain/loss and higher-moments framework, see Keating and Shadwick (2002).

Perhaps more serious estimation problems arise in the selection of representative hedge fund strategy indices. While there is some overlap in strategy coverage as demonstrated by Table 5 (only a few strategies are represented in all indices), the very same strategy is often represented by indices with significantly different statistical properties (for illustration of statistical comparison see Fawcett (2001) or Brooks and Kat (2001) for more quantitative insight). For example, Zurich Capital Markets distressed index includes many funds of hedge funds and even some equity hedge funds.

Table 5: Selected hedge fund indices mappings

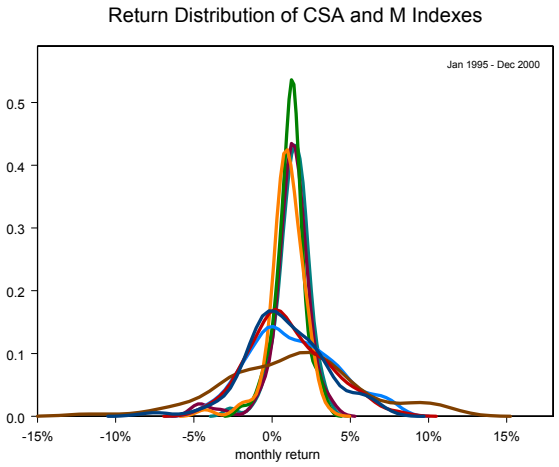
	Index	Tremont	HFR	Altvest	Hennessee
Strategy	**Code**	**CT**	**HFR**	**ALT**	**HEN**
Long Bias	**HLB**	(Yes)	Yes	(Yes)	
Hedged	**H**	(Yes)	Yes	(Yes)	(Yes)
Neutral	**MN**	Yes	Yes	(Yes)	Yes
Short bias	**S**	Yes		Yes	Yes
Event-driven	**ED**	Yes	Yes	Yes	Yes
Multistrategy	**MS**				Yes
Merger Arb	**MA**		Yes	Yes	Yes
Convertible Arb	**CSA**	Yes	Yes	Yes	Yes
Macro	**M**	Yes	Yes	Yes	Yes
Fixed Income	**CA**	Yes			Yes
Distressed	**DIST**		Yes	Yes	Yes
Emerging	**EME**	Yes			Yes
CTA	**CTA**	Yes			

Fig. 6 Recent performance of four convertible arbitrage strategy indices



Note that return distributions of indices of the **very same** hedge fund strategies are very different, for example four macro (M) and convertible arbitrage (CSA) strategies index return distributions are shown at Fig. 7.

Fig.7 Return distributions of two selected sets of hedge fund strategy indices



It is clear that numerically sensitive optimisation, factor modelling or VaR calculation procedures can be seriously affected. Therefore in practice we should rely on carefully constructed peer groups and in examples that follow below Fauchier's own "pure" strategy data is used.

As Fig. 4 shows, better characteristics can be achieved by strategic portfolios with a higher allocation to "off the shelf" fund of hedge funds and below we will examine optimised strategy allocation.

Example 1: Standard Strategy Mix Optimisation - "Return Enhancer"
Starting from the current weightings of an "off the shelf" portfolio of hedge funds the optimiser produced the following strategy mixes along the efficient frontier:

Table 6: Strategy mix efficient portfolios

	Benchmark	**Current**	Min. risk	2	3	**6**
Relative Return	0.00%	**23.51%**	0.00%	4.69%	9.37%	**23.43%**
Tracking Error	0.00%	**26.88%**	0.00%	4.03%	8.07%	**20.41%**
Absolute Return	141.00%	**164.51%**	141.00%	145.69%	150.37%	**164.43%**
Absolute Risk	140.02%	**157.74%**	140.02%	141.29%	142.67%	**147.59%**
Marginal Sharpe Ratio			999.99	1.16	1.16	**1.04**
Conf. of beating target	0.00%	**79.88%**	0.00%	81.96%	85.03%	**86.41%**
Risk of Shortfall	0.00%	**18.84%**	0.00%	11.05%	11.65%	**12.30%**
Rel. return at conf.level	0.00%	**-10.94%**	0.00%	-0.48%	-0.97%	**-2.73%**
H	12.50%	**20.85%**	12.50%	12.81%	13.12%	**14.38%**
HLB	12.50%	**24.63%**	12.50%	14.77%	17.04%	**23.96%**
EDMS	12.50%	**16.86%**	12.50%	11.99%	11.47%	**8.54%**
M	12.50%	**9.37%**	12.50%	11.75%	10.99%	**8.23%**
MN	12.50%	**8.92%**	12.50%	12.65%	12.80%	**12.91%**
MA	12.50%	**5.62%**	12.50%	12.62%	12.74%	**13.24%**
CA	12.50%	**4.15%**	12.50%	10.67%	8.84%	**5.00%**
CSA	12.50%	**9.60%**	12.50%	12.75%	13.00%	**13.75%**

Note that neutral "benchmark" portfolio is an equally weighted strategy mix.

Fig. 8 Efficient portfolios – optimal strategy mix

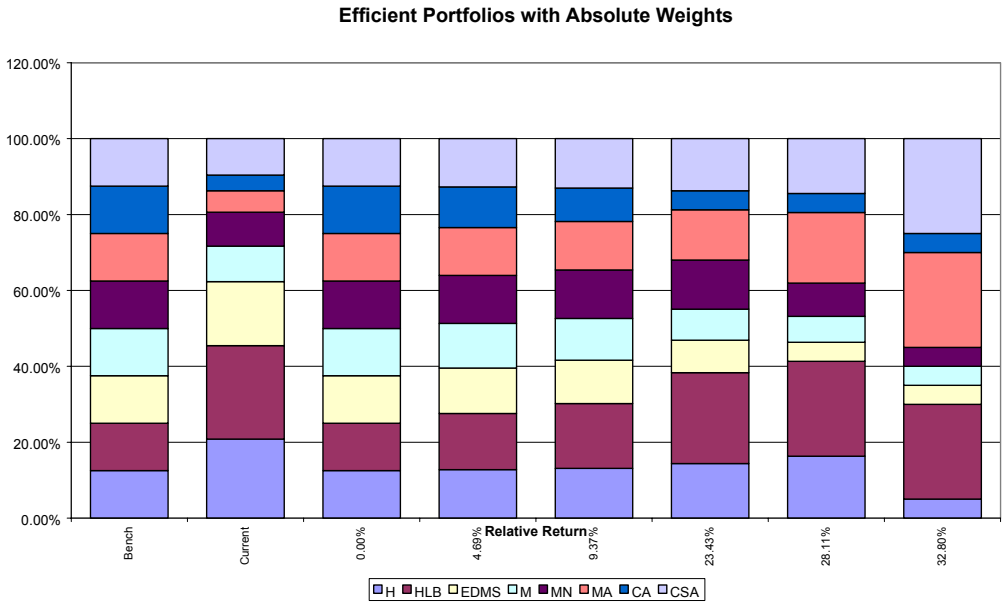
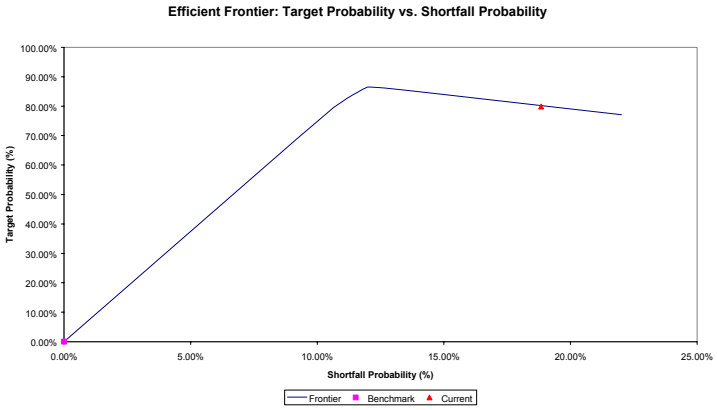


Fig. 9 An alternative representation of efficient frontier - optimal strategy mix



The graphs suggest that the current strategy mix can be improved. Assuming strategic portfolio "return enhancer" criteria, portfolio #6 offers a better risk/return profile and the suggested rebalancing action would be to swap directional and arbitrage strategy allocations (reduce H and EDMS strategy allocations and increase allocations to MA and CSA strategies).

However, an additional requirement can be to minimise the correlation of the customised portfolio of hedge funds with a selected basket of asset classes: for example, in case of a typical CEO high net worth client, with the single stock and equity holdings (i.e. "hedge" equity component of the strategic portfolio).

Example 2: Advanced Strategy Mix Optimisation - "CEO Risk Diversifier"

Consider a strategic portfolio with large equity exposure where the customisation objective is to minimise correlations with the basket of equities (Table 7).

Table 7: "CEO" Strategic Portfolio - Equity correlations

Robust, Apr 95 – Oct 00	Single Stock	SP500	MSCIXUS
Single Stock	1.00	0.50	0.49
SP500	0.50	1.00	0.69
MSCIXUS	0.49	0.69	1.00

The non-trivial problem is how to simultaneously minimise all three correlation pairs by varying the strategy mix in portfolio of hedge funds?

Note that the optimisation problem is not in standard form

$$\max U = \sum wR + \lambda wVw^T$$

Virtually all optimisers bundled with investment management tools do not allow re-definition of the optimisation objective.

However, using more flexible stand-alone optimisation engines we can solve for minimal correlation between portfolio returns S and basket B:

$$\frac{\sqrt{\sum (S-\hat{S})^2} * \sqrt{\sum (B-\hat{B})^2}}{\sqrt{\sum (S-\hat{S})(B-\hat{B})}} \leq \rho$$

in a two-step procedure: firstly a compact representation of the equity basket is obtained by principal components analysis (PCA finds a set of linear combinations (portfolios) which are orthogonal and which taken together explain all the variance of the original returns).

Statistical PCA procedure using non-normalised data (thus preserving variance of data) and robust covariance matrix is used in estimation. The principal components given in Table 8:

Table 8: Principal components of "CEO" Strategic Portfolio

PCA Components	Comp. 1	Comp. 2	Comp. 3
St. dev.	0.06	0.03	0.02
Proportion of variance	0.71	0.21	0.08
Cummulative Proportion	**0.71**	**0.92**	**1.00**

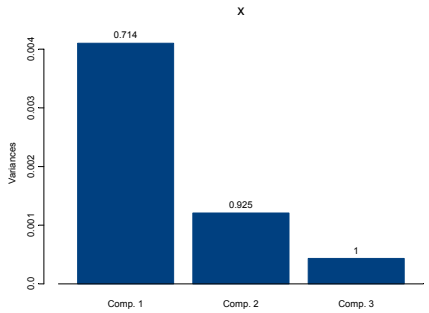
will correspond to principal vectors in Table 9:

Table 9: Principal vectors of "CEO" Strategic Portfolio

PCA Vectors	Comp. 1	Comp. 2	Comp. 3
Single Stock	0.77	0.63	0.01
SP500	0.45	-0.54	-0.71
MSCIXUS	0.45	-0.55	0.70

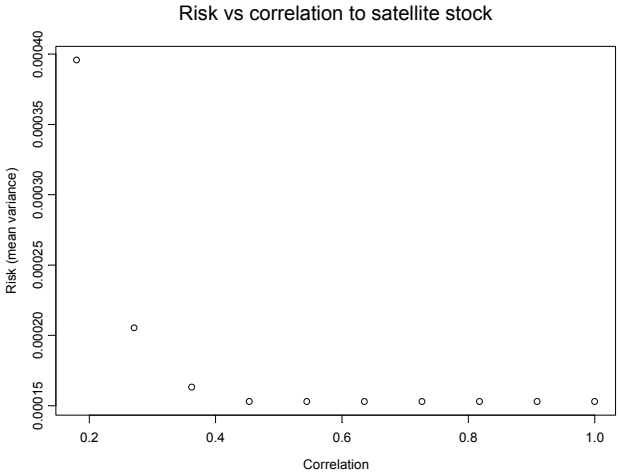
The first component (or projection) explains over 70% of variance and corresponds to a virtual "satellite stock" portfolio that will be "hedged". The principal components have changed only marginally over different time intervals indicating a stable correlation structure.

Fig. 10 Principal component variance of "CEO" Strategic Portfolio



The correlation requirement is an additional constraint that is itself optimised rather than imposed as an additive penalty term in the objective function as is the usual treatment for turnover or trading cost constraints.

Fig. 11 Correlation/Variance trade-off



The minimum correlation between the optimised strategy mix portfolio and the "satellite stock" calculated by optimiser is 0.16 corresponding to the strategy mix in Table 10.

Table 10: Minimally correlated strategy mix

Strategy	**Current**	Markowitz Optimised	Correlation Optimised
H	**20.85%**	5%	25%
HLB	**24.63%**	5%	5%
EDMS	**16.86%**	5%	5%
M	**9.37%**	5%	21.40%
MN	**8.92%**	25%	8.60%
MA	**5.62%**	23%	5%
CA	**4.15%**	6%	5%
CSA	**9.60%**	24%	25%

Conclusions

The option-like payoffs and unusual correlation profiles of hedge funds open new avenues in strategic portfolio construction. The strategy mix optimisation may be necessary to meet advanced risk requirements including minimising correlation with principal components of strategic portfolio risk or design of structured products but a flexible stand-alone optimiser may be necessary. However, liquidity and operational risk considerations may still elude quantitative analysts.

Author is grateful to Insightful UK and OCCAM Financial Technologies for their assistance.

References

Brooks, C., Kat, H. (2001) The statistical properties of Hedge Fund Index Returns and Their Implications for Investors, ISMA Centre Working Paper, U. Reading, October 2001
Fawcett, C. (2001) Hedge fund strategies, AIF Conference, FT, 28 June 2001
de las Heras, E. (2000) Understanding Private Equity, MSDW Global Pensions Quarterly, August 2000
Ineichen, A. (2000) In search of Alpha, Alternative Investment Strategies, Global Equity research, UBS Warburg, October 2000
IPD (2000) IPD Monthly Data Index, October 2000
Keating, C. and Shadwick, W. (2002) A Universal Performance Measure, Finance Development Centre, preprint
Peskin, M. et al (2000) Why Hedge Funds Make Sense, Quantitative Strategies, Global Equity and Derivative Marktes, MSDW November 2000
Schneeweis, T., Spurgin, R. and Karavas, V. (2000) Alternative investments in Institutional Portfolios, AIMA - update of 1998 study, main results are also in Schneeweis et al (2001) Understanding Hedge Fund Performance published by Lehman Brothers, November 2001
Schneeweis, T., Spurgin, R. (2000) Hedge Funds: Portfolio risk diversifiers, return enhancers or both? U. Massachusetts Technical report, July 2000
Siddiqui, T. (2000) Insurance without premium, ISI The Capital Guide to Alternative Investments
Wrighton, J. (2000) Model Investing: Tailoring a Portfolio for an Executive Fit, WSJ Europe, 1-2 December 2000
Zurich Capital Markets (2001) www.zurichcapital.com Introduction/ Additional Information, Appendix II, Fig. 11d, p. 59